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## MODES OF CONSTRUCTING INDEX-NUMBERS.

WITH the increasing use of the method of index-numbers in measuring the direction and extent of general movements in the average level of prices, the considerable divergence between the modes of construction of different index-numbers merits attention. It is very common to find the results of the computations of this or that authority referred to as giving *the* change in the cost of living. Surveying the field and noting the not inconsiderable differences between the results obtained by the different calculators, some ground for the scepticism of unsympathetic critics seems to be afforded. On the other hand, the believers in the method of index-numbers can point to the general similarity of the indications of movement which is found in spite of the divergent methods of computation. For practical purposes a wide range of variation in the material used, and in the mode of combining its different parts, is found to be admissible. Nevertheless, it would seem to be desirable to avoid, as far as possible, the most glaring deviations from methods which have stood the test of much criticism. Simplicity of preparation and freedom from mysterious technicalities tend to command confidence. Moreover, the very careful inquiries of the learned committee of the British Association for the Advancement of Science, which devoted much time to the theoretic aspects of the subject nearly twenty years ago, led to the conclusion that not much was to be gained in accuracy by the sacrifice of simplicity, provided the basis of calculation were well chosen and adequately representative.

The present paper does not aim at traversing the ground so well covered by Professor Edgeworth in the reports

to which allusion has just been made. A brief survey of some of the peculiarities of certain well-known index-numbers will, however, serve to develop some points of critical interest raised by an examination of the varying methods employed. Among the best known index-numbers are those of the London *Economist* and of Mr. Sauerbeck, both relating to English prices. Each of these is based on a comparatively limited number of representative prices, whose variations are measured by the percentage borne by these variations to the average prices of a selected period of years. The relative influence on the result which a particular group of commodities can exert is determined by the number of articles in the group, in the first place. Thus, to take a topic on which a good deal has been written, among the twenty-two price-ratios used in compiling the *Economist* number, cotton has been assigned two places, and cotton yarns and fabrics two others. Relatively to the entire list, the importance of changes in the price of cotton is somewhat exaggerated, with effects well illustrated by the high figures to which this index rose during the American Civil War. But another feature of the simple-arithmetic-average method of combining price-ratios, which is so generally used, and which appeals by its simplicity and straightforwardness, lies a little below the surface. When an important change in the relative price-levels of different groups of commodities has occurred, the equality of influence on the result which was originally accorded to the different items is capable of serious modification. As an illustration, we may take, from Mr. Sauerbeck's results, the items of Java Sugar, Olive Oil, and Straits Tin in 1904 and 1905. All of these had changed in price in the same proportion in the year's interval, and, were the original equality of importance in affecting the aggregate result able to be preserved, their indices would have moved equally. But

relative changes in price-levels had caused the three indices to register respectively 40, 64, and 121 in 1904, and 45, 72, and 136 in 1905. The original equality of influence is so modified that equal ratios of price-change affect the aggregate price-index by amounts so different as 5, 8, and 15, respectively. Even on a larger scale the same disturbing influence may be observed. Thus the 45 price-ratios which go to make up Mr. Sauerbeck's aggregate index include eight items for vegetable food, seven for animal food, and four for other food (sugar, coffee, and tea). The average indices for these three groups in the decennium 1896-1905 were 61, 82, and 50, respectively. The proportions in which equal percentage price-changes in each of the nineteen articles which are included in these three groups would influence the group totals are, therefore, approximately, 10,  $11\frac{1}{2}$ , and 4, in place of 8, 7, and 4. The course of events has increased the effective weighting, relatively to the third group, of vegetable food by 25 per cent. and of animal food by 65 per cent. It is possible that these changes of relative weighting may correspond to changes in the relative importance attaching to the groups of commodities affected, so that a desired end may be attained without the need of tinkering with the basis of the calculations. Whether it can be maintained that the changes which occur *must* reflect with approximate accuracy such modifications of relative importance of different articles seems, to say the least, doubtful. The lapse of any prolonged interval of time between the period whose prices form the basis for the entire series of numbers, and the present, is likely to involve considerable effective weighting of the items of an unweighted index-number. The actual results may not be of great importance.

To test the effect of a change in the date with reference to which the price-ratios are calculated, the writer recal-

culated the index-numbers of Mr. Sauerbeck and of the *Economist* some ten years ago,<sup>1</sup> using the data recorded by these authorities, but starting the calculation from the prices of the year 1886 instead of from the average of 1867-77, and of 1840-45, respectively. The results shown in a series of numbers extending over twenty years did not include any variations in the final averages so striking as to give much emphasis to the apparent dangers lurking in the insidious indirect weighting of the items to which reference has been made. In the case of Mr. Sauerbeck's index-number the greatest divergence in the period 1876-96 was found in the first two years. In 1876, for example, the numbers as calculated by Mr. Sauerbeck show prices 37 per cent. above the level of 1886. The recomputation, with 1886 as reference date, gave the price-level of 1876 as 41 per cent. above that of 1886. The divergence of 3 per cent. between the two indices for 1876 relative to 1886 is not so large as to shake reliance on the general indications of the index as computed by its careful compiler. With the *Economist* figures the results were not quite as accordant. Thus the price-level of 1880 is shown by the journal's own numbers as 25½ per cent. above that of 1886. The recalculation, with 1886 as starting-point, gave the price-level of 1880 as 33 per cent. above that of 1886. This divergence, being 6 per cent. of the index, is more striking; but it is the largest shown in the series 1876-1896. A test of the figures for January 1, 1907, on the original basis, and also starting from 1886 as reference-date, showed a rise of prices of 23½ per cent. in the interval since 1886 as measured by the numbers given in the *Economist's* own calculation, while the rise is nearly 29 per cent. as measured by the similar calculation on the revised basis of 1886 prices. The Aldrich Committee's report on

<sup>1</sup> Cf. *Memours and Proceedings of the Manchester Literary and Philosophical Society*, vol. xli., Part III, Session 1896-97.

wholesale prices contains some illustrations of the effect of changing the reference-date for index-numbers of the simple-arithmetic-average type, but no new reflections are suggested by them, especially as the chief changes substituted a single year for a reference-period without making any great change in the length of time intervening between the date from which calculations start and the present time.

Turning to the index-numbers calculated by Bradstreet's and Dun's, respectively, we find further curious effects of weighting, not wholly intentional, perhaps. In the case of Bradstreet's index-number we have an average based on quotations for 107 commodities. The index for each article is the cost of one pound weight. But all articles do not influence the aggregate equally. The 107 articles are divided into 13 groups, and the average price per pound of the items in each group forms the index of that group, the sum of the 13 group indices being the final index-number. In the group of metals, for example, silver is included. A little consideration will show that to average the cost of equal weights of silver, copper, iron, etc., will be, in general, to permit the fluctuations of silver to dominate the group. Even the inclusion of several quotations for iron and steel but partially offsets the handicap. On May 1, 1906, the index for the metal group was 73.83 cents. On that date silver stood at 66½ cents per *ounce*, and must clearly have overwhelming influence on a group of 14 articles only one other of which (quicksilver) was over 50 cents per pound, and which included seven iron and steel items, none of which reached 2 cents per pound. Again, in the chemical group of eleven items, at the same date, opium stands at \$2.75 per pound, and quinine at 18 cents per ounce, so that the six items in the group which stood at from 1 to 4½ cents per pound could have but little share in the group-

average index-number of 70.17 cents. A third example of extraordinary weighting in this index-number is afforded by the group of textiles, which includes eleven items. Here we find silk at \$4.25 per pound, and scoured Australian combing wool at 85 cents per pound (on the date already mentioned), while hemp and jute and flax are between 5 and 10 cents per pound, and the remaining items between these two groups. The index-number for the entire textile group, \$2.5468, is obviously likely to respond more markedly to variations in the value of silk than to those of the commoner, but not less generally important, flax or hemp or jute. On May 1, 1906, Textiles, Provisions, and Hides and Leather were the three heaviest contributors to the aggregate index, in the order named. They contributed practically two-thirds of the aggregate, while Breadstuffs, Coal and Coke, Building Materials, and Naval Stores together accounted for less than 3½ per cent. of the aggregate.

That, with such remarkably uneven and incongruous weighting, the aggregate index should yet reflect the price-changes in close correspondence with other and evenly weighted index-numbers over the last dozen years is something which would hardly have been expected, yet such is the fact.

Dun's index-number is prepared on an even more comprehensive basis than Bradstreet's. It covers 350 quotations, with due allowance for the relative importance of each.<sup>1</sup> The weighting of the several constituents of the index-number is made to correspond to the average consumption of each commodity per inhabitant of the United States, if we interpret the statement of the compilers in its apparently intended meaning. Luxuries are not included in the articles whose prices are used. So far as appears, the data for the weighting are not

<sup>1</sup> Cf. *Dun's Review*, December 7, 1901, p. 9

revised from time to time to correspond with actual changes in consumption. This revision is, it is true, a matter of very great difficulty, so much so that the weighted index-number compiled by Mr. Inglis Palgrave, using the list of articles and the prices of the *Economist*, has not been kept up to date for lack of satisfactory estimates of the consumption of the various items from year to year. A fixed scheme of weights is capable of getting as thoroughly out of touch with the facts, in the course of a long term of years, however carefully adjusted at the start, as a carelessly arranged scheme could be, and may be little, if at all, preferable to an entire absence of weighting, except in so far as the inclusion of several quotations for the more important commodities may constitute an effective rough weighting in the latter case.

Bradstreet's and Dun's index-numbers do not correspond with exactitude. The broader movements are, however, similarly indicated by each. The extent of the rise from the beginning of 1897 to the fairly steady level reached in 1903 and 1904, is shown by both at about 30 per cent. This is a somewhat greater percentage change than that indicated by the Bureau of Labor index-number, according to which this change of price-level is about 25 per cent.

This index-number of the Bureau of Labor is sufficiently familiar to readers of this journal not to need any elaborate description. It is compiled according to the methods employed by Mr. Sauerbeck, tho based on more numerous price-quotations. In place of Sauerbeck's 45 components the Bureau uses more than 250 (the number has varied between 251 and 261). Manufactured commodities as well as raw products figure largely in this list. In addition to such comment as was ventured on above in relation to all simple-arithmetic averages of price-ratios, the Bureau compilation is open to some criticism in regard to the selection of commodities, and



the degree of relative importance assigned to the various groups of articles. Thus cloths and clothing are assigned nearly 30 per cent. of the entire number of quotations, while food items, including the majority of the farm products, obtain but a little over 25 per cent. of the total number. Metals and implements and lumber and building materials together account for as many quotations as food. If the object has been to secure a representation of the different lines of expenditure in rough correspondence with their importance to the average person, it may be permitted to doubt whether that object has been fully attained. Thus eight series of quotations of meats and six of meat-yielding animals are given, while six are given of sheetings and five of shirtings, besides other cotton fabrics and cotton yarns. There are five quotations of overcoatings, five of suitings, and four of women's dress goods. Does not this overweight the clothing in comparison with the meat group? Other instances might be given to illustrate the point at issue. As already noted in other connections, however, but little change of importance is introduced, as a rule, by considerable changes in the weighting of an index-number.

There are two important questions to which one might expect to find the answer in the indications of a well-devised and carefully kept index-number. The one is, "What is the change in the money-cost of the things we buy, due to price-changes since any given date in the past?" The second is, "What is the average change in the value of money relative to other things since any given date in the past?" In one sense these are but two forms of one question. But in another sense they are different. The first presents a concrete problem: given a certain mass of commodities, what would they cost, or in what proportions would their cost vary, at different times? It will be readily seen that this differs substantially from

the question of how price-variations affect the cost of living. The former question ignores, the latter should distinctly include, considerations relating to the changes in standards of living which are developing. It is difficult to know just what many writers do mean by changes in the cost of living,—difficult to be quite certain that one's own thoughts are not fluctuating from one conception to another in relation to this form of words. If we merely desire to know what changes in the aggregate cost of a given heterogeneous mass of commodities occur from time to time, an index-number with fixed weights to its several constituents meets our requirements. But the fixing of the weights is a matter not to be lightly dismissed. If they correspond to the relative importance of different commodities at the initial date from which the calculations start, the resulting index should show the changing cost of the goods which met the needs of that day. As not a few of them become more or less obsolete, their places being taken by things some of which are of a quite different character, the index which results may not closely reflect the changing purchasing power of money after a considerable interval of years from the starting-point. If the weights are made to correspond with the relative importance of commodities at some date other than the initial date from which all price-changes are calculated, as shown above, the different commodities are affected by the degree in which their price-ratios vary from the standard (100) as well as by the multipliers selected for the purpose of weighting them. This consideration also applies to the ordinary mode of applying a changing series of weights. In fact, with changing weights it would seem desirable to carry out the series of comparisons with a fixed date by the inverse of the usual process. If the cost of a given mass of commodities, corresponding with the changing consumption year by year, be figured

out on the basis of the prices of a reference-period, expressed as percentages of those of each current year, the result will more truly express the effect of the price-changes of the intervening time on the current expenditure than if the present cost of the goods consumed at a past date were calculated in the usual way, expressing present prices as percentages of those of the said date in the past. It is far from certain that the index of price-change would be the same for the two methods named. The consumption of the past at the prices of the present and the consumption of the present at the prices of the past do not necessarily lead to equal measures of change in average prices.

A method of computing the effect of price-changes which yields results not dependent on the location of the starting-point of the calculation is to calculate each year the ratio of change from the price-level of the preceding year, and compound the resulting ratios to obtain a continuous index. This method is illustrated in a series of calculations presented in the *Economist* early in each year. The foreign trade of the British Isles is taken as the basis of the calculation, and the degrees in which changes in the volume of trade and changes in the price-level have contributed to the movement in the aggregate values recorded is determined. Thus the value which the exports of 1906 would have had if sold at the prices deduced from the record for 1905 is calculated. The comparison of this with the actual value shows how much price-change has contributed to the recorded trade-movement. It is, in effect, the calculation of an index-number of price-change for the year, the weighting being according to the importance of the different articles in the trade of 1906. It may be interesting to record the result of compounding these yearly changes for a period of thirty years to date. This is done in the following table, arranged, for convenience

of later comparisons, so that the average for the ten years 1890-99 is 100:—

BRITISH PRICE-MOVEMENT, 1876-1906.

| YEAR.      | FOREIGN TRADE PRICE-INDEX. |                  |  | Sauerbeck's Index. |
|------------|----------------------------|------------------|--|--------------------|
|            | Imports Retained           | Domestic Exports | Composite.<br>$\frac{1}{2}$ Import index +<br>$\frac{1}{2}$ Export index |                    |
| 1876 . . . | 154.4                      | 140.0            | 143 6  | 143.2              |
| 1877 . . . | 159.2                      | 134.6            | 140.8  | 142.2              |
| 1878 . . . | 146 4                      | 129.7            | 133 9  | 132.6              |
| 1879 . . . | 138.9                      | 121 2            | 125.6  | 125.8              |
| 1880 . . . | 146.6                      | 126 1            | 131.2  | 133.1              |
| 1881 . . . | 146.0                      | 119.9            | 126.4  | 128.2              |
| 1882 . . . | 143.3                      | 119.6            | 125.5  | 127.1              |
| 1883 . . . | 139.2                      | 116.0            | 121.8  | 124.5              |
| 1884 . . . | 130.2                      | 111.4            | 116.1  | 114.8              |
| 1885 . . . | 122.8                      | 106.0            | 110.2  | 108.9              |
| 1886 . . . | 115.9                      | 100 3            | 104.2  | 104.5              |
| 1887 . . . | 113.9                      | 99.8             | 103.3  | 103.6              |
| 1888 . . . | 110.8                      | 100.7            | 103 2  | 106.7              |
| 1889 . . . | 112.2                      | 103.4            | 105.6  | 109.5              |
| 1890 . . . | 110.6                      | 110.0            | 110.2  | 108.7              |
| 1891 . . . | 111.2                      | 109.0            | 109.5  | 108.6              |
| 1892 . . . | 106.8                      | 103.6            | 104 4  | 103.5              |
| 1893 . . . | 104.1                      | 101.9            | 102.4  | 103.2              |
| 1894 . . . | 95 9                       | 97.5             | 97.1   | 95.4               |
| 1895 . . . | 92.5                       | 94.1             | 93.7   | 94.1               |
| 1896 . .   | 94.2                       | 95.3             | 95.0   | 92.3               |
| 1897 . .   | 94.1                       | 94.2             | 94.2   | 93.9               |
| 1898 . .   | 94.4                       | 94 0             | 94.1   | 97.0               |
| 1899 . .   | 96.0                       | 100.3            | 99.2   | 103.2              |
| 1900 . .   | 103.9                      | 114.8            | 112.1  | 113.8              |
| 1901 . .   | 100.1                      | 108 9            | 106.7  | 106.4              |
| 1902 . .   | 99 5                       | 103.2            | 102.3  | 105.4              |
| 1903 . .   | 99.8                       | 104.4            | 103 3  | 105.4              |
| 1904 . .   | 100.0                      | 105.1            | 103 8  | 106 5              |
| 1905 . .   | 100.6                      | 105 5            | 104.3  | 109.8              |
| 1906 . . . | 104.3                      | 110.2            | 108.7  | 116.7              |

Here are shown the indices of price-movement as compiled by Mr. Sauerbeck and as determined in the manner

just described. There are given separately indices for that part of the imports not re-exported unchanged and for the exports of British and Irish produce. A composite number is also given, in which the price-changes measured by the export-index are made three times as important as those measured by the import index. The close correspondence between this and Sauerbeck's index is striking, except for the final years of the table, where both import and export indices show a lower level of price than does that of Sauerbeck.

The price-index prepared in the way just illustrated is able to be adjusted without difficulty so as to start from any desired level. Sauerbeck's index, and all those on the same basis, are, as shown earlier in this paper, affected by the starting-point. Were it recalculated so that each item stood at 100 for the period 1890-99, for which the figures in the table average 100, instead of each being at about 150 for the period 1867-77, some considerable changes might be shown. Whether the correspondence between the composite trade-index and Sauerbeck's thus modified would be greater or less than appears between the two last columns of the table cannot be determined without trial. For the trade-index, however, we know in advance that the selection of any period whatever as that for which the index is to be 100 will make absolutely no difference to the ratio of the index for any one of the years to that for any other.

Sir Robert Giffen and Mr. Bowley have determined the effect of price-changes on the figures of British foreign commerce by calculating for each of a series of years what value would be given to the trade if the prices of some one of the series were supposed substituted for those of all the rest.<sup>1</sup> They have applied directly the

<sup>1</sup> *Report to the Secretary of the Board of Trade on Recent Changes in the Prices Exports and Imports* (C-5386), 1888; *Economic Journal*, vol. vii, 1897, pp. 274-278.

method of the weighted-arithmetic average, and their results give a direct answer to one question; namely, what would the imports or exports of each year have cost at the prices of the selected reference year? They cannot give an answer to the question of what the value of the trade of any year would have been at the price-level of any but the reference year. For that the method just described serves as well as theirs. It has at least one advantage deserving of mention. When new commodities enter into the trade, the ratio of their prices to the price-level of a period when they were unknown or neglected cannot be satisfactorily determined. Either an index-number must exclude all new commodities or it must make them enter on an irregular and artificial basis if the fixed-reference-period method is followed. When the comparison which influences results is always with the preceding year, there is no difficulty about introducing new articles or dropping old ones, or substituting one series for another where standards change. A related point is that, under one and the same name, a commodity of changing character may be scheduled. To investigate the nature of the goods which bear a given designation, so as to insure identity, is more feasible when the only comparison demanded is from year to year than when comparison is needed with a distant past. If the changes are continuous and slow, there may not be much to choose between the two methods, as the changes may escape detection in either case.

The writer has experimented with the method of year-to-year steps applied to the data which yield the index-number of the Bureau of Labor, with a view to determining whether, in practice, the change of method would be a frequent cause of important changes in results. Incidentally, some other questions of considerable importance, on which the results secured might throw some light,

were kept in view. As stated earlier, the series of quotations used in the formation of the Bureau Index are numerous, varying from 251 to 261 in different years. It appeared possible, therefore, to learn something from an examination of the way in which the individual price-ratios grouped themselves on either side of the mean.

A question which soon presented itself was whether the items should be combined in the ordinary way, by taking their arithmetic average, or whether it were not proper to use the geometric average, following Jevons. When the yearly steps were determined, they would naturally be compounded geometrically, and this fact kept the question of geometric averages of the items which contribute to the determination of the yearly steps prominently in view. The choice seems to turn somewhat on our conception of the problem before us. It was stated earlier in the paper that two problems at least may be defined as offered for solution by the use of an index-number. If we want to know the money-cost of a defined group of purchasable commodities, doubling the outlay on one is not generally balanced by halving the outlay on another. But, if we conceive of our individual ratios as so many indices of the strength of the forces at work to modify the influence which money exerts on prices, a force which, acting alone, would increase the purchasing power of money by 25 per cent. needs, to balance it, a force which, acting alone, would diminish the purchasing power by, not 25, but 20 per cent. The average effect of changes must be measured by a geometric mean if we look at the matter from this standpoint, while, if we look at the matter from the other standpoint, the arithmetic average would appear to be the more appropriate to the problem. Now there cannot be much doubt that one of the things sought by students through index-numbers is a measure of the influence of money on prices, disentangled

from the influence of technical features of production, and from the causes which modify the demand for different commodities in varying degrees, and other influences which need not be defined in detail. If we approach the problem with these points occupying our thoughts, instead of with the thought of how much a given sum of money can buy or how much money is needed to buy our year's requirements, the geometric mean of the price-ratios will appear at least as natural for the solution of our problem as the harmonic or arithmetic means, which are the natural solutions, if these latter aims monopolize our thoughts.

The following table shows the magnitude of the yearly steps for fifteen years:—

PERCENTAGE OF PRICE-CHANGE FROM YEAR TO YEAR.

| DATE.               | Bureau Index. | DIRECT COMPARISON YEAR BY YEAR. |                 |
|---------------------|---------------|---------------------------------|-----------------|
|                     |               | Arithmetic Mean.                | Geometric Mean. |
| 1890-91 . . . . .   | 98.9          | 99.8                            | 99.1            |
| 1891-92 . . . . .   | 95.0          | 95.6                            | 95.1            |
| 1892-93 . . . . .   | 99.5          | 100.0                           | 99.3            |
| 1893-94 . . . . .   | 91.0          | 91.5                            | 90.9            |
| 1894-95 . . . . .   | 97.4          | 98.3                            | 97.4            |
| 1895-96 . . . . .   | 96.6          | 97.0                            | 96.1            |
| 1896-97 . . . . .   | 99.2          | 100.4                           | 99.3            |
| 1897-98 . . . . .   | 104.1         | 104.6                           | 104.1           |
| 1898-99 . . . . .   | 108.9         | 110.3                           | 108.5           |
| 1899-1900 . . . . . | 108.7         | 109.2                           | 108.5           |
| 1900-01 . . . . .   | 98.2          | 98.8                            | 97.9            |
| 1901-02 . . . . .   | 104.1         | 104.2                           | 103.6           |
| 1902-03 . . . . .   | 100.6         | 101.5                           | 101.0           |
| 1903-04 . . . . .   | 99.5          | 99.9                            | 99.2            |
| 1904-05 . . . . .   | 102.6         | 102.9                           | 102.6           |

In comparing the two central columns of this table, we can trace the effect of the constant reversion to equal weighting of the various items, which characterizes the third column. The second column, as pointed out



earlier, is practically the result of weighting more heavily changes in items which stand high relative to their starting price-level, and less heavily changes in those which stand low relative to the same standard. The geometric mean reproduces yearly steps of price-change in striking correspondence with those of the arithmetic mean thus subtly weighted.

The differences as exhibited in the above table are not great enough to arouse much interest, but the following table shows what these small variations may mean, in their cumulative effect:—

INDEX-NUMBERS BY THREE METHODS.

| YEAR.          | Bureau Index. | INDEX FROM YEAR-BY-YEAR RATIOS. |                 |
|----------------|---------------|---------------------------------|-----------------|
|                |               | Arithmetic Mean.                | Geometric Mean. |
| 1890 . . . . . | 112.9         | 109.4                           | 113.2           |
| 1891 . . . . . | 111.7         | 109.2                           | 112.2           |
| 1892 . . . . . | 106.1         | 104.3                           | 106.5           |
| 1893 . . . . . | 105.6         | 104.4                           | 105.8           |
| 1894 . . . . . | 96.1          | 95.5                            | 96.1            |
| 1895 . . . . . | 93.6          | 93.8                            | 93.6            |
| 1896 . . . . . | 90.4          | 91.0                            | 89.9            |
| 1897 . . . . . | 89.7          | 91.4                            | 89.3            |
| 1898 . . . . . | 93.4          | 95.6                            | 93.0            |
| 1899 . . . . . | 101.7         | 105.4                           | 100.9           |
| 1900 . . . . . | 110.5         | 115.2                           | 109.5           |
| 1901 . . . . . | 108.5         | 113.7                           | 107.2           |
| 1902 . . . . . | 112.9         | 118.5                           | 111.0           |
| 1903 . . . . . | 113.6         | 120.3                           | 112.2           |
| 1904 . . . . . | 113.0         | 120.2                           | 111.3           |
| 1905 . . . . . | 115.9         | 123.7                           | 114.2           |

The numbers are adjusted so that the average for the ten years 1890–99 is 100 in each case. It will be observed that the Bureau index is somewhat less accurately adjusted to this standard than the others, its error being

about one-eighth of one per cent. The figures are as stated in the Bulletin for March, 1906.

The trifling differences shown in the preceding table are seen to be productive of variations of some importance, while the correspondence of the index formed from the composition of geometric means of price-change from year to year, with that formed by the arithmetic mean of the ratios of each year's prices to those of a fixed period, is very striking. Further, as we pass from the period 1890-99, the divergences show signs of increasing.

As mentioned above, a point kept in view in the work was the grouping of the individual price-ratios about the mean. The outcome of the calculations, thus far, has not appeared susceptible of simple interpretation, tho the writer believes that points of considerable importance will receive illumination when the work is advanced far enough for effective interpretation. If we examine the individual ratios making up an aggregate index-number such as Sauerbeck's, the wide range covered is remarkable. Thus the year 1875, tho a year within the reference-period 1867-77, shows one item over 50 per cent. above the initial figure, and another over 40 per cent. below it, and there was no considerable concentration about the mean. In 1901, to take a later example, one item was nearly double the average, one differed little from one-third of the average, and these were not wholly isolated extreme variations. With so small a number of items as 45, no safe conclusion from the wide scattering can be drawn. Examining the grouping of the 259 separate price-ratios of the Labor Bureau's index for 1905, we find that, apart from an extreme item just below 40, there is fairly continuous grouping from about 60 to 200, with three items beyond 200. There is also marked concentration about the mean, more than one-half of the items being included within a range of about 12 per cent. on

either side of the mean. The modulus, calculated in the usual fashion, is rather over one-third of the mean. It is, therefore, no matter for surprise that index-numbers calculated from similar material by different methods should manifest considerable differences.

As might be expected, the modulus for the year-by-year variation is considerably smaller than for the variation of 1905 on 1890-99. In the fifteen years under observation, and for the material here examined, the modulus varied from a little over 15 per cent. of the mean to nearly 24 per cent. The lowest value was for 1904-05, the highest for 1896-97, and the average of the fifteen years was between 18 and 19 per cent. of the mean change. A comparison was made of the prices of 1902 directly with those of 1890, the geometric mean being employed. The result gave a fall of price in the twelve years of 1.4 per cent. The figures given in a preceding table represent the fall at 2 per cent., using the geometric mean of the year-to-year variations. As several changes in the commodities included had occurred in the interval, the agreement is very close. The Bureau index is the same for these two years. The component items of the comparison of 1902 direct with 1890 show much less concentration than in the case above considered, of 1905 compared with 1890-99. The modulus is nearly 50 per cent. of the mean, which expresses the wider distribution in a more precise form.

As may be supposed, the component ratios in all these comparisons are not evenly distributed on the two sides of the mean. In some cases the skewness is considerable. When a comparison is made with the standard classification given by Professor Karl Pearson in his well-known *Memoirs*, the criterion obtained does not denote that all fifteen of the year-to-year groupings belong to the same type. An attempt to apply the formulæ devel-

oped by Professor Edgeworth in his paper read to the Royal Statistical Society last year has, so far, not yielded satisfactory results.

Whether the 260 (or thereabouts) series of quotations used by the Bureau of Labor in compiling its index-number form a fair sample of the price-movements in the business of this continent, and thus give significance to results deduced from them by such theoretical methods as those indicated, is perhaps questionable. The study of price-changes of large numbers of commodities may, however, yield information worth having, even if it do not lead us to change very much our estimate of the mean movement of prices. It may be suggested that the margin of probable error in an estimate of the change in the general price-level becomes in general considerable when the interval over which the change is observed is at all prolonged.

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